

Applied mathematics, a national view

by Hirsh Cohen

The development of applied mathematics in the United States during the past forty years is described, including the role of computers and computation in enlarging and changing the field. The growth of the profession is also discussed, along with needs for training and funding for the future.

As my contribution to this celebration of mathematics in IBM Research, I would like to have a look with you at the development of applied mathematics in the United States. The perspective that I will use comes from taking part in the preparation of some of the many reports on the mathematical sciences of the past four or five years and from reading most of the other ones: the Browder Panel Report to the Office of Science and Technology Policy [1], the David Report [2], the Lax [3] and Rheinbolt [4] reports on computers and computation, reports by Olkin and Moore [5] on statistics and by Nemhauser and Dantzig [6] on operations science, and a new report that the Board of Mathematical Sciences of the NRC is working on which should appear in 1986. All of these attest to the fact that we mathematical scientists have been studying ourselves and the scientific communities with whom we work.

Before I deal with the present, however, I would like to set the scene by looking at where we were forty years ago in applied mathematics and what the world of applied

mathematics was like at that time. The choice of dates is not arbitrary, as I will explain. After I do that I will come back and try to say where we are today.

Before World War II there was virtually no profession of applied mathematics in this country. There were no degrees granted, as far as I know; there were no journals devoted to the subject. As for practitioners, William Prager of Brown University wrote "... any applied mathematics that was done was done by physicists and by engineers and for the most part mathematicians looked down on applied work ...". In 1941, Vannevar Bush, the head of the Office of Scientific Research and Development, asked Warren Weaver to start the Applied Mathematics Panel. There were very few people to draw on who were trained in any of the fields that we now think of as applied mathematics. There were some at Bell Laboratories, there were some mathematically sophisticated engineers like von Karman, von Mises, and Timoshenko. There were some pure mathematicians who joined the Applied Mathematics Panel: for example, Hassler Whitney worked on fire control problems and made important contributions to fire control and proximity devices that were used in the air warfare over England. There was Richard Courant's small crew at NYU which was just getting going at the time.

Von Karman was an important voice. He gave the Gibbs Lecture of the American Mathematical Society, "The Engineer Grapples with Nonlinear Mechanics," and he wrote the lead article for the first issue of *The Quarterly of Applied Mathematics* in 1943, "Tooling Up Mathematics for Engineering." Those were both important messages to mathematicians and engineers.

One of the key functions of the Applied Mathematics Panel was to give contracts. This was not done before World War II. It was a novel activity and it set the pattern for what the Office of Naval Research was going to begin doing in

©Copyright 1987 by International Business Machines Corporation. Copying in printed form for private use is permitted without payment of royalty provided that (1) each reproduction is done without alteration and (2) the *Journal* reference and IBM copyright notice are included on the first page. The title and abstract, but no other portions, of this paper may be copied or distributed royalty free without further permission by computer-based and other information-service systems. Permission to *republish* any other portion of this paper must be obtained from the Editor.

1945. For example, contracts enabled Courant, at NYU, to bring in people like Bernard Friedman and Max Shiffman and to begin the training of a crew of other people, whom you all know. Gene Isaacson, Joe Keller, Harold Grad, Peter Lax, and Louis Nierenberg were all part of the start-up crew.

At Brown University the summer schools in applied mathematics began in 1941. They were a response to a report that the National Research Council had prepared, led by T. C. Fry of the Bell Laboratories. Fry claimed that, with war imminent, the defense industries would need mathematicians. What was, in fact, happening with the Applied Mathematics Panel and at centers such as NYU and Brown, was that young people, for the first time in this country, were being recruited into and being trained in this new field, applied mathematics. These were, of course, not the only centers of activity. At the Radiation Laboratories at MIT, Weiner and his colleagues were starting communication theory and the mathematics of signal processing. At Los Alamos, Stanislaus Ulam, with a lot of other mathematicians, was discovering applied mathematics.

In Philadelphia and in Princeton and, in fact, at the newly formed Watson Laboratory of IBM at Columbia University (the old Watson Laboratory) there was another kind of mathematical activity going on, the beginnings of automatic, stored program computation. Von Neumann, Herman Goldstine, and others were bringing numerical analysis to the new computational machinery.

What was happening is that from almost nothing, applied mathematics was quickly getting started. The most important thing was that young people were coming into this new field. What were these young people working on? Many were doing classical applied mathematics problems, problems of the mathematics of continuum mechanics, some of which had been going on for a century. When Courant and Friedrich's book on supersonic flow and shock waves was published in 1946, it was almost 100 years since Stokes had worked on shock waves to understand coal mine explosions in the 1840s. In the classical areas the problems brought on by the war were difficult and required new methods. For example, in fluid mechanics, work was going on in blast waves; in solid mechanics, on plasticity theory. At Los Alamos and elsewhere, the need for numerical methods and, of course, the need for computation became obvious to everybody. Hyperbolic partial differential equations were a key attack point. The Monte Carlo method was invented. The handling of linear analysis numerically was recognized as a vital point of the computational approach. New methods were developed and old ideas remembered or re-invented.

There were also new problem areas: control theory, for example, was created. The electronics work that was done at the Radiation Laboratories and elsewhere called for understanding system and device control. It was started at

places like MIT and then it was blessed when mathematicians such as Lefschetz, the topologist at Princeton, picked it up and worked on it. Linear programming is probably the most striking contribution of that period (it appeared, actually, just after the war). It opened the way to computational optimization. George Dantzig's methods solved important problems almost immediately but, equally important, created a mathematical attitude toward computation that has proven to be of great value both computationally and mathematically.

Thus, what began in those years was essentially a new profession for mathematicians and other scientists in this country, new people entering that profession, and a recognition by these and by some of the older people that there was a lot to do. There were an enormous number of problems that could be attacked, and there also was a first taste of computation and the recognition that it would be essential. Let me make one more remark about this period. All of this developing period was blessed by the careful guidance of a handful of very talented Europeans.

In 1957 when Emanuel "Mannie" Piore invited Herman Goldstine to start a mathematics department here at Yorktown Heights, these beginnings that I have described were already bearing fruit. Some of the people who came here were graduates of Brown or NYU. They had come out of those starting places and in fact some were already students of the first products of those centers. Bernard Friedman had gone from Courant to Berkeley and at least one of his students, Farouk Odeh, then came here. Others had been influenced by Weiner, or Dantzig, or by Lefschetz. I will leave the story of the developments of mathematics at IBM Research to Herman Goldstine, who was, of course, responsible for it.

So much for the origins in the wartime forties. Much of what has happened in the past 40 years is the subject of the papers at this symposium and in this volume, the successes in broad areas of the mathematical sciences. In core mathematics there have been outstanding results in all of the main branches. There has been a great deal of abstraction and of unification between the branches. These accomplishments are described well in the David Report [2]. This has been a kind of golden age for mathematics. I mention this because I believe that there is now a great watershed of mathematics, waiting to be used, a mathematical potential of great power. I believe there are cyclic periods in mathematics, a turning toward abstraction and a pursuit of mathematics for its own sake followed by a turning back again to the world outside of mathematics for new stimuli, observation, and application. That second phase, the returning to the outside world, has come on again in the past five years, and it has brought new mathematical concepts and methods into many areas of application. This time, in particular, with a return to applications, there is

computation. We now have vast computational powers that were not available before. This means that we can shorten the route from the mathematics itself to its applications and in some cases make it possible.

If you agree that there is a large body of new mathematics available for applications, who is going to do the work? What has happened to the few young people that were beginning to be assembled in the 1940s? Today there are about 6000 members of the Society for Industrial and Applied Mathematics (SIAM). That is one measure of the growth of the field. There are probably several thousand more who do not belong to SIAM. So there are thousands of people now in the field who have been trained and who are practicing as applied mathematicians. There are, by the way, also applied mathematics societies in England, Germany, France, and Canada. The first joint international meeting of all these applied mathematics societies will be held in Paris in the spring of 1987.

Even if this is a good record of growth over the past forty years, it is not enough. That is, there are not enough people to do the mathematics work that has to be done. There is a shortage of applied mathematicians. The ability to compute has increased. The penetration of mathematics into many fields has increased. I counted the number of applied mathematics theses in the listing of Ph.D. theses in mathematics for 1985. There are altogether about seven hundred in all fields of mathematics. Only about ten percent were in applied mathematics, not counting statistics. Of those in applied mathematics, seventy or eighty, those who have specialized in first-rate numerical work come to about a dozen. The effect of this kind of production rate is that at places like IBM Research, the Bell Laboratories, and all of the best university departments who try to hire the top young people in the field, the same young people have been circulating around through the interviewing process. There is a shortage in that field, numerical mathematics, just as trying as the better-known shortages in computer science and VLSI work.

Now, applied mathematics is not a very well-known field. I think that is one of the causes of this shortage I just mentioned. Young people do not even know enough to ask about whether their schools have curricula in that field. The lucky ones find out. Young people do not know what has been accomplished in and by applied mathematics. They do not know, for example, that there is mathematics in the design of the Boeing 767 wing, or in CAT scans, or in chip design, or in oil recovery. If there is going to be a transfer of the great wealth of mathematics that has accumulated, and if the array processors, vector processors, parallel processors, and the millions of PCs are going to be used effectively for science, engineering, and design, we will need more mathematically trained people. This, in fact, has been one of the main political messages that the applied mathematicians have been trying to deliver in the past few years of

mathematical political activity. All those many reports I mentioned as I started have called for more funds for research, more computers and computation time, and especially more support for graduate students and postdoctoral work. When the Browder Panel started its work in 1982, there were 50 federally supported postdoctoral positions in mathematics compared to 1200 in physics and 2500 in chemistry. The idea of having postdoctoral assignments in mathematics was not a very popular one, but it is one that does work well in applied mathematics. Due to the David Report and the activity it engendered, the number of postdoctoral positions has gone up rapidly in the past three years' time. There are several hundred now and I think it is going to go up further. Similarly, the number of federally supported graduate students has been going up rapidly. This is part of a general rise in funding for mathematical sciences and especially for applied mathematics. Of the \$105 000 000 that has been requested in the 1987 federal budget for the mathematical sciences (this does not include computer sciences, but does include statistics), I would estimate that about \$40 000 000 is for applied mathematics. That is a healthy percentage, and you can compare it with the number of people being turned out in the graduate schools to do that work.

During this past five-year period when all this introspection and soliciting has been done, it has been interesting to me to see that in many cases, and in many circumstances, applied mathematics has been able to take the lead, for a very simple reason. It is easier to talk about the applications of mathematics, easier than talking about mathematics itself. If you are to talk to the head of OTSP, the Office of Technology and Science Policy, who was a physicist, or the head of NSF, who is an engineer, or the undersecretary for Research and Engineering Development of DOD, who was an aeronautical engineer, or to the undersecretary for Research for the Department of Energy, who is a plasma physicist, then it is good to talk about applications. They are readily grasped and there is no doubt that they are valuable. So it has turned out that we applied mathematicians can help by taking the lead. It also has turned out that we have recognized that there is an "old boy" system working for us. Not just old boys in the mathematical community, whom we know something about, but over the past 40 years we have made a lot of friends in other parts of the scientific world. In fact, all four of those agency heads that I mentioned are either friends or friends of friends of somebody in mathematics, and that has been very helpful. That is not a new trick in science policy or science funding. Our colleagues in physics and chemistry and elsewhere have known about it for a long time.

Let me briefly explain my view of what the strategy of the mathematics community has been in the past four years' time. At the beginning, when the Browder Panel started and the David Report was getting under way, it was quite clear

to everybody that funding for the kinds of things I have talked about, especially for young people, was at such a low ebb that what we required was a general health and welfare injection for the mathematical sciences. That is, in fact, what the David Report calls for, a doubling of the funding for the mathematical sciences in five years' time. That is about 15 percent a year, and in the first two and maybe three years that has been achieved. But after the first successes this health and welfare argument wears out. The obvious next phase, the next initiative, has to be related to more specific areas of contribution by the mathematical sciences. The obvious choice is the mathematics of computation. Defined in the broadest sense, this goes all the way from hard-core numerical methods to the kinds of computations that are used by analysts and algebraists and geometers in doing their own mathematics. This would be the second major vehicle to keep that 15 percent climb going on.

This kind of suggestion, however, gives rise to an inevitable debate in the mathematics community. What happens if you do not get the funding for the new initiative, but have made it sound so good that you've been asked to do it out of old funds? Will the other parts of the mathematical sciences be left behind? Anybody who has worked in an industrial research laboratory for some years understands this situation. It would be nice to pull everything along evenly, but you cannot do that; something has to lead. I believe that the mathematics community should take the risk of putting forward an initiative like the mathematics of computation. It is equivalent in physics to the large particle generators, or for the astronomers, their telescopes. It is the big machine idea. We do not have a claim to all the computers and all the computation that everybody does, but we have the rights of ancient domain to the mathematics of computation. We should step forward with it. This kind of initiative has been discussed quite thoroughly with people at NSF in a year of budget deficits, and there already is some movement. I do not know how it will fare in the long run.

We have indeed come a long way since the 1940s when all of those young people, new to the field, were being trained. We are a small but established profession in America. However, we are not well enough known, either to prospective students whom we need, or for our accomplishments. The successes, as I have said, are still hidden. In spite of this, I think we are in a position to lead the mathematical sciences to whole new areas of endeavor because of this closeness that we have always had to computation. We need to be sure, as we do this, that we form very strong alliances with the computer sciences and to be sure, as new areas of that discipline arise, that we recognize them and stay close to them. Above all, we have to keep in mind that we have always thrived, as applied mathematicians, on being the bridge between mathematics and the rest of the civilized world.

References

1. *Report of the Research Briefing Panel on Mathematics* (COSEPUP/NAS); and the DOD Addendum to this Report, National Academy Press, Washington, DC, 1983.
2. *Renewing U.S. Mathematics, Critical Resource for the Future*, National Academy Press, Washington, DC, 1984.
3. *Report of the Panel on Large-Scale Computing in Science and Engineering*, Peter D. Lax, Chairman (DOD and DOE in cooperation with NSF and NASA), National Science Foundation, 1982.
4. *Computational Modeling and Mathematics Applied to the Physical Sciences*, National Academy Press, Washington, DC.
5. David S. Moore and Ingram Olkin, "Statistics: Change and Resources in a Growing Science" (report to NSF Mathematical Sciences Advisory Committee), *Amer. Statist.* **38**, 1 (February 1984).
6. George Nemhauser and George Dantzig, "Operations Sciences at NSF: Status and Opportunities," *Proceedings of Workshop on Research Directions in Operations Science*, internal NSF document.

Received September 26, 1986; accepted for publication November 11, 1986

Hirsh G. Cohen *IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598.* Dr. Cohen received his B.S. in applied mathematics from the University of Wisconsin, Madison, in 1947 and his M.S. and Ph.D. in applied mathematics from Brown University, Providence, Rhode Island, in 1948 and 1950, respectively. He spent the period from 1950 to 1959 teaching and doing research in a number of universities in the United States and in other countries. He joined the Mathematical Sciences Department of the IBM Thomas J. Watson Research Center in 1959. Dr. Cohen served as Director of Mathematical Sciences from 1967 to 1968 and became IBM Assistant Director of Research in 1969. He acted as Director of the IBM Zurich Research Laboratory in 1974 and 1981. He became Director of Yorktown Laboratory Operations in 1981 and Vice President, Divisional Operations, in 1985. During these years, he also, at various times, held visiting professorships at the California Institute of Technology, Pasadena, at the Hebrew University of Jerusalem, Israel, and at the Cornell Medical Center, New York, New York. Dr. Cohen has served in a number of government advisory and professional society positions, including President of the Society for Industrial and Applied Mathematics, 1983-1984. He is currently a member of the Air Force Office of the Scientific Research Advisory Committee; the Board of Mathematical Sciences, National Research Council; the Energy Research Advisory Board, Department of Energy; and the Mathematical Sciences Education Board, National Research Council. His research interests include nonlinear ordinary and partial differential equations, fluid dynamics, vibration analysis, nonlinear diffusion, problems in mathematical biology, and how to make research laboratories work.